Approximate $\sqrt{21}$ by using a linear approximation to $f(x) = \sqrt{10x - x^2}$ based at $x_0 = 2$. Use differentials.

(note: $f(2) = 4$ & $f(3) = \sqrt{21}$)

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$$\sqrt{21} = f(3) = f(2+1)$$

$$f(x+\Delta x) \approx f(x+dx)$$

$$f(x+dx) = f(x) + dy$$
$$= f(x) + f'(x)\,dx$$
$$= f(x) + f'(x)(x - x_0)$$

$$f'(x) = \frac{1}{2}\frac{1}{\sqrt{10x-x^2}}(10-2x)$$
$$= \frac{5-x}{\sqrt{10x-x^2}}$$

$$\sqrt{21} \approx f(2+dx)$$
$$= f(2) + f'(2)\,dx$$
$$= 4 + \frac{3}{4}(3-2)$$
$$= 4\frac{3}{4}$$

$$\therefore \sqrt{21} \approx 4\frac{3}{4}$$